Find all octants in which $xz < 0$ and $y > 0$ simultaneously. **SCORE: ____ / 10 PTS**

2 $x > 0, y > 0, z < 0 \rightarrow O_{1+4}$ OR O_5 $3\frac{1}{2}$

2 $x < 0, y > 0, z > 0 \rightarrow O_2$ $2\frac{1}{2}$

Let P be the point $(-2, 4, -1)$, R be the point $(1, 6, -2)$, and $\overrightarrow{PQ}$ be the vector $-5\vec{i} - \vec{j} + 4\vec{k}$. **SCORE: ____ / 103 PTS**

3 EACH EXCEPT AS NOTED

[a] Find the coordinates of Q.

$\langle x+2, y-4, z+1 \rangle = \langle -5, -1, 4 \rangle$

$\begin{cases} x+2 = -5 \\ y-4 = -1 \\ z+1 = 4 \end{cases}$ $(x, y, z) = (-7, 3, 3)$

[b] Find the area of triangle PQR.

$\overrightarrow{PR} = \langle 3, 2, -1 \rangle$

$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -1 & 4 \\ 3 & 2 & -1 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -5 & -1 \\ 3 & 2 \end{matrix} = \langle -7, 7, -7 \rangle$

$\frac{1}{6}$

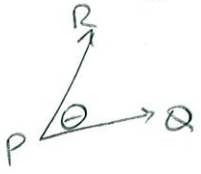
CHECK: $\langle -7, 7, -7 \rangle \cdot \langle 3, 2, -1 \rangle$
$= -21 + 14 + 7 = 0$
$\langle -7, 7, -7 \rangle \cdot \langle -5, -1, 4 \rangle$
$= 35 - 7 - 28 = 0$ ✓

$\frac{1}{2} \| \langle -7, 7, -7 \rangle \| = \frac{1}{2} |-7| \| \langle 1, -1, 1 \rangle \|$

$= \frac{1}{2} \cdot 7 \cdot \sqrt{3}$

$= \frac{7\sqrt{3}}{2}$

[c] Find $\angle RPQ$.



$\cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \cos^{-1} \frac{-15 - 2 - 4}{\sqrt{42}\sqrt{14}} = \cos^{-1} \frac{-21}{\sqrt{42}\sqrt{14}}$

$= \cos^{-1} \frac{-3 \cdot 7}{\sqrt{6}\sqrt{7}\sqrt{7}\sqrt{2}}$

$= \cos^{-1} \frac{-3}{2\sqrt{3}}$

$= \cos^{-1} \frac{-\sqrt{3}}{2} = \frac{5\pi}{6}$ OR $150°$

CONTINUED ON NEXT PAGE ➡

⬅ CONTINUED FROM PREVIOUS PAGE

[d] If $\overrightarrow{PR}$ is parallel to $< -5, 2-c, b+1 >$, find the value of b.

$\langle 3, 2, -1 \rangle = k\langle -5, 2-c, b+1 \rangle$ 4

$= \langle -5k, (2-c)k, (b+1)k \rangle$

$3 = -5k$ $\quad -1 = (b+1)k$

$k = -\frac{3}{5}$ $\quad -1 = -\frac{3}{5}(b+1) \longrightarrow b = \frac{2}{3}$

[e] Find a vector of magnitude 9 perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

$$\frac{9}{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|}(\overrightarrow{PQ} \times \overrightarrow{PR}) = \frac{9}{7\sqrt{3}}\langle -7, 7, -7 \rangle$$

$$= 3\sqrt{3}\langle -1, 1, -1 \rangle$$

$$= \langle -3\sqrt{3}, 3\sqrt{3}, -3\sqrt{3} \rangle \text{ OR } \langle 3\sqrt{3}, -3\sqrt{3}, 3\sqrt{3} \rangle$$

[f] Find the general **(NOT point-normal)** equation of the plane which contains P, Q and R.

$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \langle -7, 7, -7 \rangle$ OR $\langle 1, -1, 1 \rangle$ 6

$(x+2) - (y-4) + (z+1) = 0$

$x - y + z + 7 = 0$

+3 SIMPLIFY

[g] Find symmetric equations of the line which is perpendicular to the plane in part [f], and also contains Q.

$\vec{d} = \vec{n}$ $\quad \frac{x+7}{1} = \frac{y-3}{-1} = \frac{z-3}{1}$ 6

$x + 7 = 3 - y = z - 3$

[h] Find parametric equations of the line which is parallel to the line in part [g], and also is perpendicular to the plane in part [f], and also contains R.

$\vec{d}_2 = \vec{d} = \vec{n}$

$x = 1 + t$

$y = 6 - t$

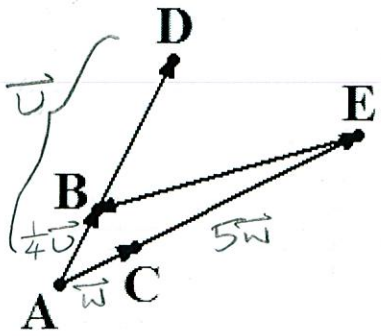
$z = -2 + t$

6

In the diagram below, ABD and ACE are both line segments. **SCORE: ____ / 10 PTS**

CE is five times the length of AC, and AD is four times the length of AB. **(NOTE: The diagram is NOT drawn to scale.)**

If $\vec{u} = \overrightarrow{AD}$ and $\vec{w} = \overrightarrow{AC}$, find an expression for $\overrightarrow{EB}$ in terms of $\vec{u}$ and $\vec{w}$.



$\overrightarrow{EB} = \overrightarrow{EA} + \overrightarrow{AB}$

$= -6\vec{w} + \frac{1}{4}\vec{u}$ OR $\frac{1}{4}\vec{u} - 6\vec{w}$

5 5

Fill in the blanks. List all correct answers. **SCORE: ____ / 12 PTS**

[a] The equation of the $xy-$plane is $z = 0$ and the equation of the $y-$axis is $x = z = 0$.

[b] If $\vec{u} \cdot \vec{u} = 10$, then $\|\vec{u}\| =$ $\sqrt{10}$ and $\vec{u} \times \vec{u} =$ $\vec{0}$.

[c] If you start at the point $(-1, -5, 1)$, then move 3 units downward, 8 units forward and 6 units to the left, you will be at the point $(7, -11, -2)$. $(-1+8, -5-6, 1-3)$

2 EACH

$\langle 0, 3, -2 \rangle = \langle 4-x, -1-y, -8-z \rangle$

[d] If the terminal point of $\vec{v} = 3\vec{j} - 2\vec{k}$ is $(4, -1, -8)$, then the initial point of $\vec{v}$ is $(4, -4, -6)$.

Consider the sphere $x^2 + y^2 + z^2 + 12x + 14y - 8z + 65 = 0$. **SCORE: ____ / 15 PTS**

[a] Find the equation of the $yz-$trace. Describe the $yz-$trace.

$x^2 + 12x + 36 + y^2 + 14y + 49 + z^2 - 8z + 16 = -16 + 36 + 49 + 16$

$(x+6)^2 + (y+7)^2 + (z-4)^2 = 36$

1½ $x = 0 \rightarrow$ $36 + (y+7)^2 + (z-4)^2 = 36$

$(y+7)^2 + (z-4)^2 = 0$

POINT $(0, -7, 4)$

2 EACH EXCEPT AS NOTED

[b] Find the equation of the $xz-$trace. Describe the $xz-$trace.

1½ $y = 0 \rightarrow (x+6)^2 + 49 + (z-4)^2 = 36$

$(x+6)^2 + (z-4)^2 = -13$

NO TRACE